Partial Identification Under Multiple Nest Structures

Qifan Han

Boston University Econometrics Seminar, Spring 2023

Outline

Nested Logit Model

- Nested logit model is one of the widely applied discrete choice model because of it captures various substitution patterns.
  - This is crucial to overcome the problem due to IIA property in simple logit models.
  - $P(\text{car}) = P(\text{red bus}) = \frac{1}{2} \Rightarrow$
    $P(\text{car}) = P(\text{red bus}) = P(\text{gray bus}) = \frac{1}{3}$ (**?**)
- Other choices: random coefficients/IPDL (Fosgerau et al., 2021), etc.
  - Nested logit model has an appealing interpretation.
  - Easy to implement with statistical packages.

Nested Logit Model

- Main Issue: to use the nested logit model, the researcher needs to define a nest structure before fitting the data.

- Specification of nest structure can play an important role:
  - Understanding the “correct" market structure: type-primary vs. brand-primary (not actually sequential)
  - Leading to a refinement of assortment planning and pricing decisions (Kök and Xu, 2011)

- There are mainly two ways to deal with concerns about nest structure:
  - Testing IIA property, following Hausman and McFadden (1984) and Vijverberg (2011)
  - Data-driven methods to identify the nest structure: Aboutaleb et al. (2008), Almagro and Manresa (forthcoming), Hortacsu et al. (forthcoming)

Nested Logit Model

- This project aims to explore the fitness between data and nested logit model while allowing for heterogeneity in nest structures.
- It follows a partial identification framework which is close to that from Barseghyan et al.(2021).
- It tries to provide the sharp identification region based on the classic Artstein's inequality between the observed conditional choice probabilities and the capacity functional based on potential nest structures under consideration.

Nested Logit Model

I try to answer the following questions from this project:

- How to propose a robust method of discrete choice analysis while being more agnostic to potential nest structures?
- Is it possible to identify all “possible" potential nest structures from a dataset with discrete choices, or figure out which one may be the most likely?
- When a nest structure is identified by some data-driven methods, how to interpret it?

Related Literature

- **Nested Logit Model**
  - Ben-Akiva (1973), McFadden (1978)
  - **Specifications:** Kannan and Wright (1991), Goldberg (1995)
  - **Consumer heterogeneity:** Berry, Carnall and Spiller (1996)
- **Nest Structures:**
  - **Identification:** Kovach and Tsenrenjigmid (2022), Almagro and Manresa (forthcoming), Hortacsu et al. (forthcoming)
  - **Test:** Hausman and McFadden (1984), Rivers and Vuong (2002)
- **Partial Identification**
  - **Methololology:** Barseghyan et al.(2021), Andrews and Shi (2013)
  - **Heterogeneity in Consumers' Preferences:** Beresteanu and Rigotti (2021)

Model

- Consider a population of individuals $\mathcal{I} = 1, 2, \ldots, I$ and a set of choices $\mathcal{J} = 1, 2, \ldots, J$.
- **Utility Function** I assume that each individual $i \in \mathcal{I}$ makes her decision $d_i = j$ from the universal choice set $\mathcal{J}$ according to the following inequality:

$$d_i = j \in \mathcal{J} \Leftrightarrow U_i(j) \geq U_i(c) \text{ for all } c \in \mathcal{J}$$

- Each individual i is characterized by a real-valued vector of observed attributes $\boldsymbol{x}_i$ and a real-valued vector of unobserved attributes $\boldsymbol{\nu}_i$.
- A random sample of $\{d_i, \boldsymbol{x}_i\}_{i \in \mathcal{I}}$ is observed.

Assumptions on Utility Function

Assumption 1:

1. There exists a function $W : \mathcal{X} \times \mathcal{V} \mapsto \mathbb{R}$, known up to a finite-dimensional parameter vector $\boldsymbol{\delta} \in \Delta \subset \mathbb{R}^k$, where Δ is convex and compact, and continuous in each of its arguments such that
$$U_i(c) = W(\boldsymbol{x}_{icrm}, \boldsymbol{\nu}_i; \boldsymbol{\delta}), \quad \forall c \in \mathcal{J}$$
2. Denote F the distribution of $\boldsymbol{\nu}_i$, which is continuous and known up to a finite-dimensional parameter vector $\gamma \in \Gamma \subset \mathbb{R}^\ell$, where Γ is convex and compact.

Model: Nest Structures

- Define N a **nested partition** of the universal choice set $\mathcal{J}$, whereas N is a family of sets satisfying the following conditions:
  1. $\varnothing \notin N$
  2. $\forall A, B \in N, A \neq B \Rightarrow A \cap B = \varnothing$
  3. $\bigcup_{A \in N} A = \mathcal{J}$
- Denote the union of all nest partitions of choice set $\mathcal{J}$ as $\mathcal{N}_{\mathcal{J}}$.
- For a particular $\mathcal{J}$, we may explicitly give an order to the nested partitions, denoted by $N_t, t = 1, \ldots, T$.

Model: Nest Structures

- e.g. If $\mathcal{J} = \{\text{car}, \text{red bus}, \text{gray bus}\}$, we can assume that

$$N_1 = \{\{\text{car}\}, \{\text{red bus}\}, \{\text{gray bus}\}\}$$
$$N_2 = \{\{\text{car}, \text{red bus}\}, \{\text{gray bus}\}\}$$
$$N_3 = \{\{\text{car}\}, \{\text{red bus}, \text{gray bus}\}\}$$
$$N_4 = \{\{\text{car}, \text{gray bus}\}\}, \{\text{red bus}\}\}$$
$$N_5 = \{\{\text{car}, \text{gray bus}, \text{red bus}\}\}$$

- In practice, researchers can only consider a subset of $\mathcal{N}_{\mathcal{J}}$, and call it "the consideration set of nest structures", denoted by $\mathcal{N}^*$.

Model

- Denote $d_i^*(\boldsymbol{x}_i, \boldsymbol{\nu}_i(N_t); \boldsymbol{\delta})$ the model-implied optimal choice for agent i given $(\boldsymbol{x}_i, \boldsymbol{\nu}_i)$, and utility parameter $\boldsymbol{\delta}$.
- Define the set of model-implied optimal choices given $(\boldsymbol{x}_i, \boldsymbol{\nu}_i)$ and parameter $\boldsymbol{\delta}$, over the consideration set of nest structures $\mathcal{N}^*$ as

$$D^*(\boldsymbol{x}_i, \boldsymbol{\nu}_i; \boldsymbol{\delta}) = \bigcup_{N_t \in \mathcal{N}^*} \{d_i^*(\boldsymbol{x}_i, \boldsymbol{\nu}_i(N_t); \boldsymbol{\delta})\}$$

Sharp Identification Region

Proposition 1

Let $\boldsymbol{\theta} = [\delta; \gamma]$, $\Theta = \Delta \times \Gamma$, *whereas the dissimilarity parameter(s)* $\boldsymbol{\lambda}_{N_t}$ *is a subvector of* γ. *The sharp identification region for* θ *is:*

$$\Theta_I = \left\{ \boldsymbol{\theta} \in \Theta : P\Big(d \in J \mid \boldsymbol{x}\Big) \leq P\Big(J \cap D^*(\boldsymbol{x}, \boldsymbol{\nu}; \boldsymbol{\delta}) \neq \varnothing; \boldsymbol{\gamma} \mid \boldsymbol{x}\Big) \right\}$$

for all $J \in \mathcal{J}$.

Simulation (Grid Search)

- 3 Alternatives: $\mathcal{J} = \{\{\text{car}\}, \{\text{red bus}, \text{gray bus}\}\}$ with dissimilarity parameter $\lambda = 0.5$.
- 3 Attributes:
  - $x_{j1} = 1\{\text{gray}\}$
  - $x_{ij2} \sim$
    $$\begin{cases} \text{Binom}(0.5) \text{ with values } 0.5 \text{ and } 1 & \text{if } j \in \{\text{red bus}, \text{gray bus}\} \\ \text{Binom}(0.5) \text{ with values } 1 \text{ and } 1.5 & \text{if } j \in \{\text{car}\} \end{cases}$$
  - $p_{ij} \sim$
    $$\begin{cases} \text{Binom}(0.5) \text{ with values } 0.5 \text{ and } 0.75 & \text{if } j \in \{\text{red bus}, \text{gray bus}\} \\ \text{Binom}(0.5) \text{ with values } 1.5 \text{ and } 2 & \text{if } j \in \{\text{car}\} \end{cases}$$
- $u_{ij} = \alpha p_{ij} + \boldsymbol{x}'_{ij}\boldsymbol{\beta} + \tilde{\epsilon}_{ij}$, $\alpha = -1$, $\boldsymbol{\beta} = (0.5, 1)'$.
- $I = 1000$, $\text{nrep} = 500$.

Identified Set: No Heterogeneity in DGP

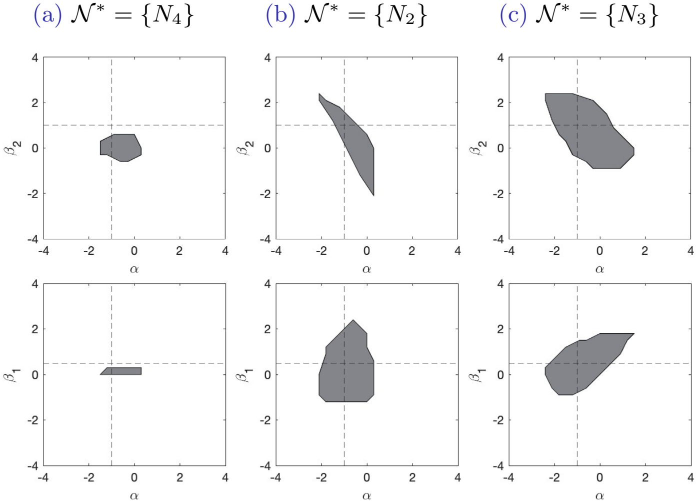
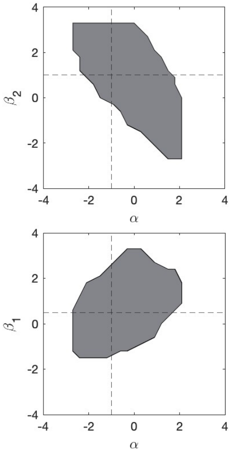


Figure 1: Identified Set: DGP with Heterogeneity

Notes: $N_2 = \{\{\text{car}, \text{red bus}\}, \{\text{gray bus}\}\}$, $N_3 = \{\{\text{car}\}, \{\text{red bus}, \text{gray bus}\}\}$,
$N_4 = \{\{\text{car}, \text{gray bus}\}\}, \{\text{red bus}\}\}$

Identified Set: No Heterogeneity in DGP

(a) $\mathcal{N}^* = \{N_2, N_3\}$ (b) $\mathcal{N}^* = \{N_1, N_3\}$



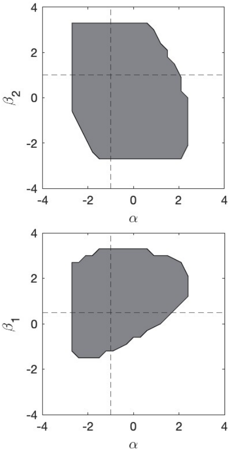


Figure 2: Identified Set: Simple DGP with Non-singleton $\mathcal{N}^*$

Notes: $N_1 = \{\{\text{car}\}, \{\text{red bus}\}, \{\text{gray bus}\}\}$, $N_2 = \{\{\text{car}, \text{red bus}\}, \{\text{gray bus}\}\}$,
$N_3 = \{\{\text{car}\}, \{\text{red bus}, \text{gray bus}\}\}$

Identified Set: DGP with Heterogeneity

- Let 50% of individuals draw $\tilde{\epsilon}_{ij}$ based on the nest partition $N_2 = \{\{\text{car}, \text{red bus}\}, \{\text{gray bus}\}\}$, and the other 50% draw from the nested partition $N_3 = \{\{\text{car}\}, \{\text{red bus}, \text{gray bus}\}\}$.

Identified Set: DGP with Heterogeneity

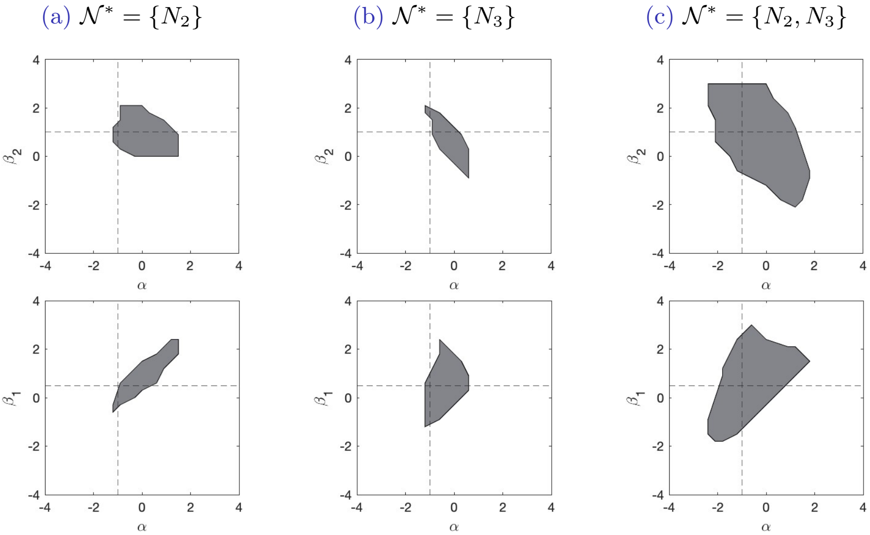


Figure 3: Identified Set: DGP with Heterogeneity

Summary

- This project tries to propose a robust method for discrete choice analysis while allowing for heterogeneity in nest structures.
- In general, it still requires some prior knowledge on the substitution pattern to further restrict the consideration set of nest structures.
- Following works:
  - Incorporating Random Coefficient Nested Logit (RCNL) model in the theoretical framework.
  - Find a suitable empirical application, ideally on identifying some relevant functionals of parameters of interest.